1. (a) Find the value of $\frac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2 \sqrt{(5x - 1)}.$$

(6)

(b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x.

(4)
(Total 10 marks)

2. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(Total 6 marks)

3. Differentiate, with respect to x,

(a) $e^{3x} + \ln 2x$,

(3)

(b) $(5 + x^2)^{\frac{3}{2}}$

(3)
(Total 6 marks)

4. A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T °C, t minutes after it enters the liquid, is given by

$$T = 400\,e^{-0.05t} + 25, \ t \geq 0.$$

(a) Find the temperature of the ball as it enters the liquid.

(1)

(b) Find the value of t for which $T = 300$, giving your answer to 3 significant figures.

(4)

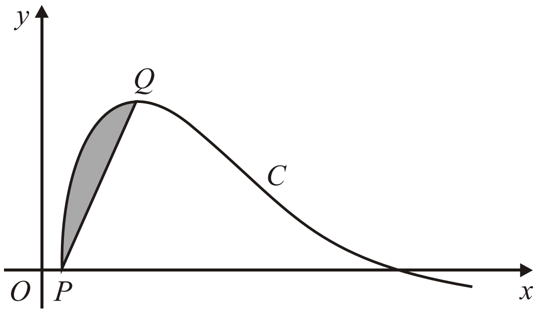
(c) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in °C per minute to 3 significant figures.

(3)

(d) From the equation for temperature T in terms of t, given above, explain why the temperature of the ball can never fall to 20 °C.

(1)
(Total 9 marks)

5.



The figure above shows a sketch of part of the curve C with equation

$$y = \sin(\ln x), \qquad x \geq 1.$$

The point Q, on C, is a maximum.

(a) Show that the point $P(1, 0)$ lies on C.

(1)

(b) Find the coordinates of the point Q.

(5)

(c) Find the area of the shaded region between C and the line PQ.

(9)
(Total 15 marks)

6. (a) Differentiate with respect to x

(i) $3 \sin^2 x + \sec 2x$,

(3)

(ii) $\{x + \ln(2x)\}^3$.

(3)

Given that $y = \dfrac{5x^2 - 10x + 9}{(x-1)^2}$, $x \neq 1$,

(b) show that $\dfrac{dy}{dx} = -\dfrac{8}{(x-1)^3}$.

(6)
(Total 12 marks)

7. Use the derivatives of cosec x and cot x to prove that

$$\frac{d}{dx}\left[\ln(\operatorname{cosec} x + \cot x)\right] = -\operatorname{cosec} x.$$

(Total 3 marks)

8. Differentiate with respect to x

(i) $x^3 e^{3x}$,

(3)

(ii) $\dfrac{2x}{\cos x}$,

(3)

(iii) $\tan^2 x$.

(2)

Given that $x = \cos y^2$,

(iv) find $\dfrac{dy}{dx}$ in terms of y.

(4)
(Total 12 marks)

1. (a) $\frac{\text{d}}{\text{d}x}(\sqrt{(5x-1)}) = \frac{\text{d}}{\text{d}x}\left((5x-1)^{\frac{1}{2}}\right)$

$= 5 \times \frac{1}{2}(5x-1)^{-\frac{1}{2}}$ M1 A1

$\frac{\text{d}}{\text{d}x} = 2x\sqrt{(5x-1)} + \frac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$ M1 A1ft

At $x = 2$, $\frac{\text{d}y}{\text{d}x} = 4\sqrt{9} + \frac{10}{\sqrt{9}} = 12 + \frac{10}{3}$ M1

$= \frac{46}{3}$ Accept awrt 15.3 A1 6

(b) $\frac{\text{d}}{\text{d}x}\left(\frac{\sin 2x}{x^2}\right) = \frac{2x^2 \cos 2x - 2x \sin 2x}{x^4}$ M1 $\frac{\text{A1} + \text{A1}}{\text{A1}}$ 4

Alternative

$\frac{\text{d}}{\text{d}x}(\sin 2x \times x^{-2}) = 2\cos 2x \times x^{-2} + \sin 2x \times (-2)x^{-3}$ M1 A1 + A1

$2x^{-2} \cos 2x - 2x^{-3} \sin 2x = \left(\frac{2\cos 2x}{x^2} - \frac{2\sin 2x}{x^3}\right)$ A1 4

[10]

2. $x = \cos(2y + \pi)$

$\frac{dx}{dy} = -2\sin(2y + \pi)$ M1 A1

$\frac{dy}{dx} = -\frac{1}{2\sin(2y+\pi)}$ Follow

through their $\frac{dx}{dy}$ A1ft

before or after substitution

At $y = \frac{\pi}{4}$, $\frac{dx}{dy} = -\frac{1}{2\sin\frac{3\pi}{2}} = \frac{1}{2}$ B1

$y - \frac{\pi}{4} = \frac{1}{2}x$ M1

$y = \frac{1}{2}x + \frac{\pi}{4}$ A1 6

[6]

3. (a) $\frac{dy}{dx} = 3e^{3x} + \frac{1}{x}$ B1M1A1 3

B1 $3e^{3x}$

M1: $\frac{a}{bx}$

A1: $3e^{3x} + \frac{1}{x}$

(b) $\left(5 + x^2\right)^{\frac{1}{2}}$ B1

$\frac{3}{2}\left(5 + x^2\right)^{\frac{1}{2}}.2x = 3x(5 + x^2)^{\frac{1}{2}}$ M1 for $kx(5 + x^2)^m$ M1 A1 3

[6]

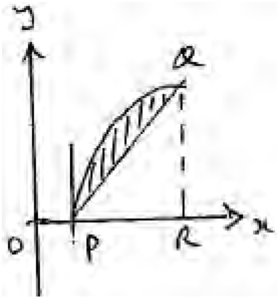
4. (a) 425 °C B1 1

	(b)	$300 = 400e^{-0.05t} + 25 \quad \Rightarrow 400e^{-0.05t} = 275$	M1	
		sub. T = 300 and attempt to rearrange to $e^{-0.05t} = a$*, where* $a \in Q$		
		$e^{-0.05t} = \frac{275}{400}$	A1	
		M1 correct application of logs	M1	
		$t = 7.49$	A1	4
	(c)	$\frac{dT}{dt} = -20\ e^{-0.05t}$ (M1 for $ke^{-0.05t}$)	M1 A1	
		At $t = 50$, rate of decrease = (±) 1.64 °C / min	A1	3
	(d)	$T > 25$, (since $e^{-0.05t} \to 0$ as $t \to \infty$)	B1	1

[9]

5.	(a)	$x = 1$; $y = \sin(\ln 1) = \sin 0 = 0$		
		$\therefore$ P = (1, 0) and P lies on C	B1 c.s.o.	1
	(b)	$y' = \frac{1}{x}\cos(\ln x)$	M1, A1	
		$y' = 0$ at $Q \quad \therefore \cos(\ln x) = 0 \quad \therefore \ln x = \frac{\pi}{2}$	M1	
		$x = e^{\frac{\pi}{2}}$		
		$\therefore Q = \left(e^{\frac{\pi}{2}}, \sin(\ln e^{\frac{\pi}{2}})\right)$	A1	
		$= (e^{\frac{\pi}{2}}, 1)$	A1	5

(c)



Area = $\int_1^{e^{\frac{\pi}{2}}} \sin(\ln x)\mathrm{d}x$ – Area $\Delta\, PQR$ (correct approach) M1

Area $\Delta\, PQR$ = $\frac{1}{2} \times 1 \times (e^{\frac{\pi}{2}} - 1)$ B1

for integral; let $\ln x = u \quad \therefore x = e^u$ (substitution) M1

$\frac{1}{x}dx = du \quad \therefore dx = e^u du$

$\underline{\mathrm{F}} = \int_0^{\frac{\pi}{2}} \sin u.(e^u du)$ A1

$= \left[e^u \sin u\right]_0^{\frac{\pi}{2}} - \int e^u \cos u du$ M1

$= e^{\frac{\pi}{2}} - \left[e^u \cos u\right]_0^{\frac{\pi}{2}} - \int e^u \sin u \mathrm{d}u$ M1

$\therefore 2\mathrm{I} = e^{\frac{\pi}{2}} + 1$

$\mathrm{I} = \frac{1}{2}(1 + e^{\frac{\pi}{2}}) = 1$ (I) A1

$\therefore$ Area = $\frac{1}{2}(1 + e^{\frac{\pi}{2}}) - \frac{1}{2}(-1 + e^{\frac{\pi}{2}}) = 1$ A1

[9]

6. (a) (i) $6\sin x \cos x + 2\sec 2x \tan 2x$ M1 A1 A1 3
or $3 \sin 2x + 2 \sec 2x \tan 1x$

[M1 for 6 sinx]

(ii) $3(x + \ln 2x)^2(1 + \frac{1}{x})$ B1 M1 A1 3

[B1 for $3(x + \ln 2x)^2$]

(b)	Differentiating numerator to obtain $10x - 10$	B1	
	Differentiating denominator to obtain $2(x - 1)$	B1	
	Using quotient rule formula correctly:	M1	
	To obtain $\frac{dy}{dx} = \frac{(x-1)^2(10x-10) - (5x^2 - 10x + 9)(2(x-1)}{(x-1)^4}$	A1	
	Simplifying to form $\frac{2(x-1)[5(x-1)^2 - (5x^2 - 10x + 9)}{(x-1)^4}$	M1	
	$= -\frac{8}{(x-1)^3}$ (*)	c.s.o. A1	6
	Alternatives for (b)		
	Either Using product rule formula correctly:	M1	
	Obtaining $10x - 10$	B1	
	Obtaining $-2(x - 1)^{-3}$		
	To obtain $\frac{dy}{dx} = (5x^2 - 10x + 9)\{-2(x - 1)^{-3}\} + (10x - 10)(x - 1)^{-2}$	A1cao	
	Simplifying to form $\frac{10(x-1)^2 - 2(5x^2 - 10x + 9)}{(x-1)^3}$	M1	
	$= -\frac{8}{(x-1)^3}$ (*)	c.s.o. A1	6
	Or Splitting fraction to give $5 + \frac{4}{(x-1)^2}$	M1 B1 B1	
	Then differentiating to give answer	M1 A1 A1	6

[12]

7.	$\frac{dy}{dx} = \frac{1}{\operatorname{cosec} x + \cot x}(-\operatorname{cosec} x \cot x + -\operatorname{cosec}^2 x)$ Full attempt at chain rule	M1	
	$= -\operatorname{cosec} x \frac{(\cot x + \operatorname{cosec} x)}{\operatorname{cosec} x + \cot x}$ Factorise cosec x	M1	
	$= \underline{-\operatorname{cosec} x}$ (*)	A1 cso	3

[3]

8. (i) $u = x^3 \quad \frac{du}{dx} = 3x^2$

$v = e^{3x} \quad \frac{dv}{dx} = 3e^{3x}$

$\frac{dy}{dx} = 3x^2 e^{3x} + x^3 3e^{3x}$ or equiv — M1 A1 A1 — 3

(ii) $u = 2x \quad \frac{du}{dx} = 2$

$v = \cos x \quad \frac{dv}{dx} = -\sin x$

$\frac{dy}{dx} = \frac{2\cos x + 2x\sin x}{\cos^2 x}$ or equiv — M1 A1 A1 — 3

(iii) $u = \tan x \quad \frac{du}{dx} = \sec^2 x$

$y = u^2 \quad \frac{dy}{du} = 2u$

$\frac{dy}{dx} = 2u \sec^2 x$ — M1

$\frac{dy}{dx} = 2 \tan x \sec^2 x$ — A1 — 2

(iv) $u = y^2 \quad \frac{du}{dy} = 2y$

$x = \cos u \quad \frac{dx}{du} = -\sin u$ — M1

$\frac{dx}{dy} = -2y \sin y^2$ — A1

$\frac{dy}{dx} = \frac{-1}{2y \sin y^2}$ — M1 A1 — 4

[12]

1. This proved a good starting question which tested the basic laws of differentiation; the chain, product and quotient laws. Almost all candidates were able to gain marks on the question. In part (a), most realised that they needed to write $\sqrt{(5x-1)}$ as $(5x-1)^{\frac{1}{2}}$ before differentiating. The commonest error was to give $\frac{\mathrm{d}}{\mathrm{d}x}\left((5x-1)^{\frac{1}{2}}\right)=\frac{1}{2}(5x-1)^{-\frac{1}{2}}$, omitting the factor 5. It was disappointing to see a number of candidates incorrectly interpreting brackets, writing $(5x-1)^{\frac{1}{2}}=5x^{\frac{1}{2}}-1^{\frac{1}{2}}$. Not all candidates realised that the product rule was needed and the use of $\frac{\mathrm{d}}{\mathrm{d}x}(uv)=\frac{\mathrm{d}u}{\mathrm{d}x}\times\frac{\mathrm{d}v}{\mathrm{d}x}$ was not uncommon. Part (b) was generally well done but candidates should be aware of the advantages of starting by quoting a correct quotient rule. The examiner can then award method marks even if the details are incorrect. The commonest error seen was writing $\frac{\mathrm{d}}{\mathrm{d}x}$ (sin2x) = cos2x. A number of candidates caused themselves unnecessary difficulties by writing sin 2x = 2sin x cos x. Those who used the product rule in part (b) seemed, in general, to be more successful than those who had used this method in other recent examinations.

2. This proved a discriminating question. Those who knew what to do often gained all 6 marks with just 4 or 5 lines of working but many gained no marks at all. Although there are a number of possible approaches, the most straightforward is to find $\frac{\mathrm{d}y}{\mathrm{d}x}$, using the chain rule, and then invert $\frac{\mathrm{d}y}{\mathrm{d}x}$ to obtain $\frac{\mathrm{d}x}{\mathrm{d}y}$. Substituting $y=\frac{\pi}{4}$ gives the gradient of the tangent and the equation of the tangent can then be found using $y - y_1 = m\,(x - x_1)$ or an equivalent method. However, many confused $\frac{\mathrm{d}y}{\mathrm{d}x}$ with $\frac{\mathrm{d}x}{\mathrm{d}y}$. Those who knew the correct method often introduced the complication of expanding cos(2y + π) using a trigonometric addition formula. Such methods were often flawed by errors in differentiation such as $\frac{\mathrm{d}}{\mathrm{d}y}(\sin\pi)=\cos\pi$ Among those who chose a correct method, the most frequently seen error was differentiating cos (2y + π) as – sin (2y + π).

An instructive error was seen when candidates changed the variable y to the variable x while inverting, proceeding from $\frac{\mathrm{d}x}{\mathrm{d}y}=-2\sin(2y+\pi)$ to $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2\sin(2x+\pi)}$. This probably reflected a confusion between inverting, in the sense of finding a reciprocal, and the standard method of finding an inverse function, where the variables x and y are interchanged.

3. Part (a) was done well by many candidates. However, as was noted in the reports on both of the previous C3 papers, some candidates have difficulty in differentiating ln(ax) ; the most common errors on this occasion being $\frac{1}{2x}$ or $\frac{2}{x}$. The chain rule was well understood and many candidates scored full marks in part (b), although a few lost the final mark because they did not fully simplify their solution. Inappropriate applications of the product rule were occasionally seen in both parts of this question.

4. Calculator work was generally accurate in this question and it was encouraging to see most candidates give their answers to the required degree of accuracy. The vast majority of candidates gave the correct answer of 425°C in part (a). Many candidates were able to substitute T = 300 in part (b) and correctly change an equation of the form $e^a = b$ to $a = \ln b$ Weaker candidates showed a lack of understanding of logarithms by failing to simplify their initial equation to the form $e^a = b$ and using an incorrect statement of the form $a = b + c \Rightarrow \ln a = \ln b + \ln c$ Not all candidates understood the need to differentiate in part (c) and found the gradient of a chord instead of finding $\frac{dT}{dt}$. The most common error made by candidates who did differentiate was to give the differential as – 20 $te^{-05.0t}$ Candidates often had difficulty giving precise explanations in part (d). Although many referred to the +25 term in their answers, far fewer gave adequate reasons as to why this meant that the temperature could never fall to 20° C, particularly with regard to $e^{-05.0\ t} > 0$. Lack of understanding of the concept of limit led some to write (in words or symbols) $T \geq 25$ rather than $T > 25$.

5. This was the question in which many candidates earned their highest marks. It was also the one for which most S marks were gained. Virtually all candidates scored the first mark. Differentiation was generally good in part (b) and many candidates scored all 5 of these marks. A common error was to state that ln x=1. There were also many good attempts at part (c). Nearly all recognized the need to take the difference of two areas. Those who sought to find the area of the triangle by forming the equation of the line and then integrating usually came unstuck in a mass of algebra and they rarely obtained the correct value. Fortunately most simply used half the base x height! Integration of y was usually well done. Similar numbers of candidates used direct integration by parts ($x$sin(ln x) etc.) as used the substitution u=ln x, resulting ine^u sin u du. Many were able to complete the two cycles of parts and obtain the correct answer.

6. (a) (i) Most candidates demonstrated good knowledge of trigonometric differentiation but there were a number of errors particularly in the derivative of sec 2x.

(ii) Candidates did not always apply the function of a function rule, but the most common error seen in this part was d/dx(ln2x) = 1/2x or 2/x. Expanding using the binomial theorem prior to differentiation was rarely seen and when used, often contained inaccuracies.

(b) Knowledge of product and quotient rules was good but execution sometimes poor. There was a lack of sustained accuracy in algebra manipulation and much alteration to obtain the answer on the paper. Most candidates did not factorise out the (x-1) factor until the last line of the solution. There were however a few excellent solutions using the division method. Many replaced solutions to this part were inserted later in the answer book, and candidates are advised to make clear reference to such replaced solutions (with a page reference) in their original solution.

7. This was an uncomfortable starter for most candidates, the derivatives of cosec$x$ and cot$x$ are, of course, given in the formula booklet but some did not seem aware of this. Many realised that the chain rule was required and gave a correct first step, but the factor of cosec$x$ was rarely spotted and often a promising start was spoilt by poor cancelling. Others tried to differentiate the function without the chain rule and the expressions $\frac{1}{\operatorname{cosec} x + \cot x}$ and $\frac{-1}{\operatorname{cosec} x + \operatorname{cosec}^2 x}$ were frequently seen.

8. No Report available for this question.